

Behavior of Triple Langmuir probes in Non-equilibrium Plasmas

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A model of current collection in Langmuir probes is used to investigate the effect representative non-equilibrium plasmas under various conditions have on the electron temperature and number density that would be calculated through analysis of the probe collection characteristics. The model uses the distribution function to calculate the charged particle flux to a probe and then, for fixed applied voltages, the current values that satisfy continuity in the probes are determined. The triple probe is not scanned in voltage, so there is no practical way using experimental triple probe data to determine if the plasma is in equilibrium. As a consequence, a triple probe analysis typically relies on the assumption that the plasma is in equilibrium. Proceeding from this point, the numerically-generated non-equilibrium triple probe data are analyzed assuming that the plasma is in equilibrium, with the data compared to the initial distribution function inputs of plasma temperature and number density to determine the effect the non-equilibrium distribution has on plasma measurements. The temperature and number density are both significantly affected when a fraction of the particles in the distribution are shifted from the equilibrium configuration into the non-equilibrium part of the distribution function. For all instances studied, the computed electron temperature and number density are extremely sensitive to small deviations from equilibrium ($\leq 5\%$ of the plasma shifted into the non-equilibrium function). Shifting more of the plasma into the non-equilibrium distribution beyond this initial level does not produce a significant additional shift in the computed plasma properties.

I. Introduction

LANGMUIR probes¹ are diagnostic tools that collect current from a plasma as a function of applied voltage to measure the electron temperature, number density, and plasma potential. Single, double, and triple Langmuir probes are commonly used in plasma diagnostics because of their relative simplicity. In the single probe, a swept voltage is applied between the probe tip and circuit common to acquire a waveform showing the collected current as a function of applied voltage. A double Langmuir probe consists of two tips, both inserted into the plasma, with a voltage applied between them. Again, a current-voltage characteristic is produced as the voltage between the probes is swept.

The triple probe² has three tips inserted into the plasma, with each tip electrically coupled to the others by non-swept applied voltages. The selected voltages represent three points on the single Langmuir probe I-V curve. Elimination of the voltage sweep makes it possible to use triple Langmuir probes to measure time-varying plasma properties in transient plasmas. Triple Langmuir probe measurements have been widely employed for various types of plasmas, including pulsed and time-varying plasmas such as those seen in pulsed plasma thrusters (PPTs),³⁻⁵ dense plasma focus devices,⁶ plasma flows,⁷ and fusion experiments.⁸

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The typical Langmuir probe analysis for determining electron temperature and number density of the plasma (for a single, double, or triple Langmuir probe) includes an assumption that the plasma is in thermal equilibrium. While this assumption may be justified for some applications, it is unlikely that it is fully justifiable for pulsed and time-varying plasmas or for the entire time a plasma device is in use. Furthermore, the swept-voltage nature of the single and double probe provides more data to the user that can be used to determine if a plasma is in equilibrium. The triple probe does not have this option and users are deprived of a significant amount of data as the measurements are all acquired for specific applied voltages only. An equilibrium analysis of experimental triple probe data in the literature is always employed because, unlike the swept single or double probes, it is difficult if not impossible for triple probe data obtained at fixed voltages to show if the plasma is in an equilibrium state.

In the present work, we return to basic governing equations of probe current collection and compute the current that would be collected by the probes when inserted into an equilibrium plasma or one of two variants of a non-equilibrium plasma. The first non-equilibrium distribution function variant consists of two Maxwellian distributions with different temperatures (the two-temperature Maxwellian). A second variation modeled involves a second Maxwellian with low temperature that is offset from zero (in velocity space) to add a suprathermal beam of electrons to the tail of the main Maxwellian distribution (the bump-on-the-tail distribution function). For a range of parameters in these non-Maxwellian distributions, we compute the current collection to the probes. We compare the distribution function that was assumed *a priori* with the plasma density and temperature one would calculate through the application of standard equilibrium triple probe theory in the analysis of the collected currents,² illustrating the effect a non-Maxwellian plasma has on the resulting triple probe measurements. The purpose of the present work is to show that the results of a triple probe analysis conducted under the assumption of an equilibrium electron distribution can deviate from the actual plasma parameters when the distribution function actually has a non-equilibrium configuration.

An outline of the remainder of this paper is as follows. In Sect. II we present various models for charge particle collection by conductors in plasmas for ions, Maxwellian electrons, and the representative non-Maxwellian electron distributions used in this work. A review of the current collection models as a function of applied voltage for single, double, and triple Langmuir probes is presented in Sect. III, including analytical results that can be derived for electrons having a Maxwellian equilibrium distribution function. Calculated triple probe data are presented in Sect. IV to show the effect non-equilibrium electron distribution functions have on the results of a triple probe analysis undertaken assuming that the plasma is Maxwellian.

II. Current Collection Models

A conductor in a plasma will collect current due to the fluxes of electrons and ions to its surface. These fluxes change as the potential applied to the conductor is varied. They will also vary for different electron distribution functions. In this section we describe equilibrium and specific non-equilibrium electron distribution function models used for the present study, and show how the electron and ion fluxes to a conducting probe are calculated as a function of applied probe voltage.

A. Equilibrium Electron Collection

In the equilibrium case, the electrons of a plasma assume a Maxwellian probability distribution function, which is shown graphically in Fig. 1a and written as a function of velocity v in one dimension as

$$f(v) = \sqrt{\frac{m_e}{2\pi k_B T_e}} \exp\left(-\frac{m_e v^2}{2k_B T_e}\right) \quad (1)$$

where m_e is the mass of an electron, k_B is Boltzmann's constant, and T_e is the electron temperature. This distribution function integrated over all velocities is equal to unity. For a conducting wall to which a voltage V has been applied, the flux of electrons to the wall (for any distribution function) is given by

$$\phi(V) = n_e \int_{v_b}^{\infty} v f(v) dv \quad \text{for } V < 0 \quad (2a)$$

$$\phi(V) = n_e \int_0^{\infty} v f(v) dv \quad \text{for } V \geq 0 \quad (2b)$$

where n_e is the electron number density and the lower integral bound v_b represents the minimum velocity an electron must possess to overcome the potential barrier presented by the plasma sheath surrounding the wall. Equating potential

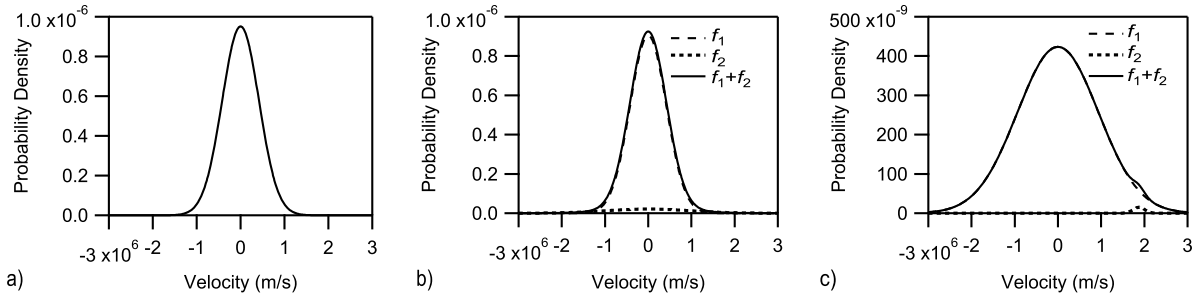


Figure 1. a) Maxwellian distribution function of Eq. (1) for $T_e=1$ eV. b) Two-Temperature Distribution of Eqs. (5) for $p=0.05$, $T_{e1}=1$ eV, and $T_{e2}=5$ eV. c) Suprathermal beam distribution for $p=0.05$, $T_{e1}=5$ eV, $T_{e2}=0.1$ eV, and $v'=10$ eV.

to kinetic energy, this velocity can be written as a function of voltage as

$$v_b = \sqrt{-\frac{2eV}{m_e}}. \quad (3)$$

where e is the elementary charge of an electron. (Some texts write the voltage as the difference between the applied voltage and the plasma potential. In the present work, our convention is to take the plasma potential as the voltage $V = 0$.) The electron current collected by the probe as a function of voltage is then

$$I_e = e A_p \phi(V) \quad (4)$$

where A_p is the probe area.

B. Non-Equilibrium Electron Collection

There are a number of different non-equilibrium electron energy distributions that we could use for the purposes of our analysis. However, in all cases the integral over all velocity space of the distribution function (or functions) should be unity, the flux to the probe as a function of applied voltage is always given by Eq. (2), and the electron current to the probe is given as Eq. (4). To keep the present work simple, we define two representative, illustrative non-equilibrium probability distribution functions to gain insight into the more general issue of the response of triple Langmuir probes used in non-equilibrium plasmas. We proceed with a description of these two distribution functions.

1. Two-Temperature Distribution

The “two-temperature Maxwellian” probability distribution function has a population of electrons in a Maxwellian distribution of temperature T_{e1} and a second population of electrons in a different Maxwellian distribution with a temperature T_{e2} . Mathematically, these two distributions can be written as:

$$f_1(v) = p \sqrt{\frac{m_e}{2\pi k_B T_{e1}}} \exp\left(-\frac{m_e v^2}{2k_B T_{e1}}\right) \quad (5a)$$

$$f_2(v) = (1-p) \sqrt{\frac{m_e}{2\pi k_B T_{e2}}} \exp\left(-\frac{m_e v^2}{2k_B T_{e2}}\right) \quad (5b)$$

where p is the fraction of electrons in the first distribution and $(1-p)$ is the fraction in the second. The value of p is bound such that $0 < p \leq 1$, and we observe that if p is equal to 1, we recover the single temperature Maxwellian distribution written in Eq. (1). The combination of the two distributions in Eq. (5) comprise the so-called two temperature Maxwellian, illustrated graphically in Fig. 1b.

2. Maxwellian with a Suprathermal Beam

This electron probability distribution function consists of the sum of a Maxwellian population of electrons and a suprathermal beam of electrons. The suprathermal beam is offset from zero velocity by an amount v' corresponding

to the beam energy, and it often has a very narrow spread of velocities. The summed distribution is often called a ‘bump-on-the tail’ distribution function for the visual effect the beam has on the overall form of the distribution. There are several ways to mathematically create a suprathermal beam, but for the purposes of this paper we shall treat it as a second, offset Maxwellian distribution. The distribution function for the population of electrons in a Maxwellian distribution of temperature T_{e1} and a suprathermal beam are written as:

$$f_1(v) = p \sqrt{\frac{m_e}{2\pi k_B T_{e1}}} \exp\left(-\frac{m_e v^2}{2k_B T_{e1}}\right) \quad (6a)$$

$$f_2(v) = (1 - p) \sqrt{\frac{m_e}{2\pi k_B T_{e2}}} \exp\left(-\frac{m_e (v - v')^2}{2k_B T_{e2}}\right) \quad (6b)$$

where the latter equation describes the probability distribution in the beam and p has the same definition as previously found in Eq. (5). When the ‘beam’ has a narrow spread in velocities (for example, electrons emitted from a nearly monoenergetic source), the value of T_{e2} , representing the spread in the velocities, is a small number. A distribution function comprised of a Maxwellian with a suprathermal beam of electrons given by Eq. (6) is illustrated graphically in Fig. 1c.

C. Ion Collection

In the seminal triple Langmuir probe work by Chen and Sekiguchi,² the ion current to a probe was assumed constant for any applied voltage while the electron current varied with voltage. In the present work, we shall maintain that assumption so as to eliminate that source of variation from our simulations and permit an evaluation of how non-equilibrium electron distributions affect the electron collection rates and commensurate plasma properties that we would calculate from triple probe data.

For cases where the electron temperature is significantly greater than the ion temperature, the ion saturation current is given by the Bohm ion current.⁹ This can be written as

$$I_i = e n_e A_p \sqrt{\frac{k_B T_e}{m_i}} \exp\left(-\frac{1}{2}\right) \quad (7)$$

where m_i is the ion mass and the term under the square root is the Bohm velocity, which is dependent on the electron temperature and the ion mass.

III. Langmuir Probe Theory Review

In the present section we proceed with a brief review of Langmuir probe theory and closed-form results obtained when the electron distribution function is assumed to be that given by Eq. (1). We also use the non-equilibrium distributions described in the previous section to show how the current collected by a single or double Langmuir probes changes as the values in Eqs. (5) or (6) are varied.

A. Single Langmuir Probe

The single probe is the most familiar and simple implementation of the Langmuir probe. In a symmetric single Langmuir probe, shown schematically in Fig. 2a, the voltage applied to the probe is swept to yield a characteristic current-voltage (I - V) curve like that shown schematically in Fig. 3. We observe that the curve has numerous features. The ion and electron saturation current levels occur at voltages where all electrons incident to the probe are either repelled or collected, respectively. The floating potential ϕ_f is the voltage where the ion and electron currents are exactly equal. The plasma potential V_p is the voltage at which the probe enters the electron saturation regime, physically corresponding to a condition where there is no longer a sheath surrounding the probe to retard the electron flow to the surface. For all probes analyzed in this paper, the probe area was assumed to be $9.42 \times 10^{-6} \text{ m}^2$.

All Langmuir probes can be thought of as single probes. Double and triple Langmuir probes, for example, are simply single probes that are electrically connected in such manner to constrain the current and/or voltage applied to the individual probes. However, these more complicated probes still operate on some portion of the single probe I - V curve, so understanding how a non-equilibrium distribution of electrons can affect the response of a single probe

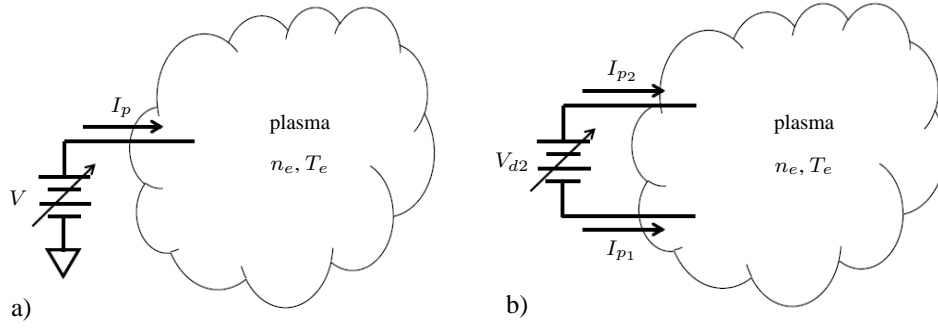


Figure 2. Schematic representation showing a) a single Langmuir probe and b) a double Langmuir probe.

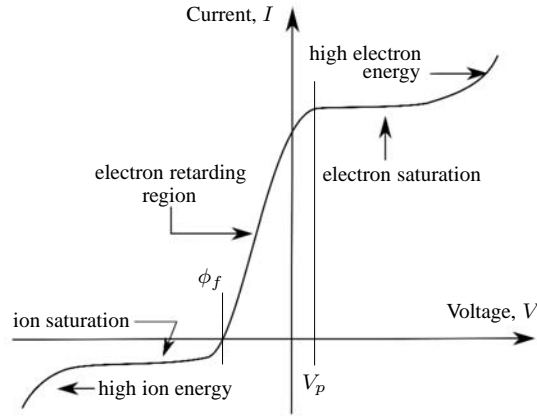


Figure 3. Schematic representation of a typical single Langmuir probe I - V curve.

will be insightful when we present and discuss the results of our double and triple probe modeling in the following sections.

A single Langmuir probe inserted in the plasma will collect a probe current I_p that is the sum of the electron and ion currents:

$$I_p = I_e - I_i \quad (8)$$

under the assumption that the direction of positive current is away from the probe surface. If the electron distribution function is a Maxwellian, then the electron flux of Eq. (2) can be integrated in closed form to yield

$$I_p(V) = eA_p n_e \sqrt{\frac{k_B T_e}{2\pi m_e}} \exp\left(-\frac{e(-V)}{k_B T_e}\right) - I_i = i_e \exp\left(-\frac{e(-V)}{k_B T_e}\right) - I_i \quad \text{for } V < 0 \quad (9a)$$

$$I_p(V) = eA_p n_e \sqrt{\frac{k_B T_e}{2\pi m_e}} - I_i = i_e - I_i \quad \text{for } V \geq 0 \quad (9b)$$

where the exponential is the Boltzmann factor that represents the repulsion of electrons in the distribution that have an energy lower than V . A numerically-generated I - V curve was obtained by integrating Eq. (2)a with $n_e = 10^{19} \text{ m}^{-3}$ and $T_e = 1 \text{ eV}$, which produced the electron flux to the probe as a function of voltage that was used in Eq. (4). The resulting I - V characteristic is plotted in Fig. 4a. The analytical solution of Eq. (9) is not shown in the figure because it would lie on top of the numerical solution.

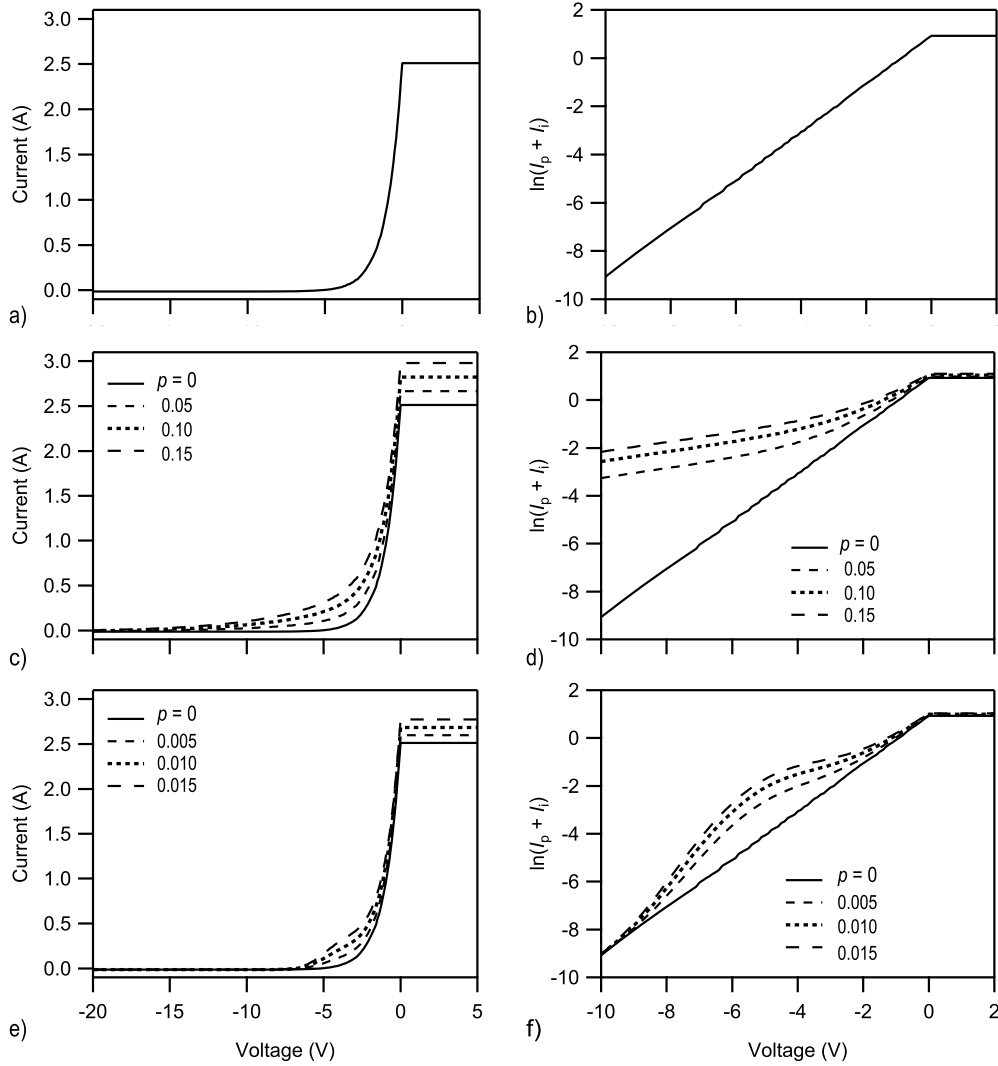


Figure 4. Single Langmuir probe I - V characteristics and $\ln(I_p + I_i)$ against V for a plasma with $n_e=10^{19} \text{ m}^{-3}$ calculated for a) and b) a Maxwellian distribution with $T_e=1 \text{ eV}$, c) and d) a two-temperature distribution with $T_{e1}=1 \text{ eV}$, $T_{e2}=5 \text{ eV}$, and p as given in the figure, and e) and f) a suprathermal beam with $T_{e1}=1 \text{ eV}$, $T_{e2}=0.1 \text{ eV}$, $v'=5 \text{ eV}$, and p as given in the figure.

Going a step further, we can rearrange Eq. (9a) as

$$\ln(I_p + I_i) = \frac{e}{k_B T_e} V + \ln(i_e) \quad (10)$$

We observe that the slope of this curve, graphed in Fig. 4b, can be used to calculate the electron temperature of the Maxwellian distribution function. Using that value and the measured ion saturation current, Eq. (7) can be used to calculate the number density n_e . Calculating the values of T_e and n_e using this method returned the original values used in the distribution function to generate the initial I - V curves.

If the electrons have a non-equilibrium energy distribution function, then the I - V curve cannot be written in closed form and must be generated by numerically integrating Eqs. (2) to yield the electron flux to the probe as a function of voltage. Once that is done, Eq. (4) can be employed to give the electron current as a function of voltage. In Fig. 4c we plot I - V curves for a two-temperature distribution function where $n_e=10^{19} \text{ m}^{-3}$, $T_{e1}=1 \text{ eV}$, $T_{e2}=5 \text{ eV}$, and p varies from 0-15%. The second, hotter population of electrons serves to increase the current, making the bend in the characteristic much more gradual for increasing p . We also observe in Fig. 4d that the slope of the $\ln(I_p + I_i)$ versus V curves for the two-temperature distribution becomes much less steep for more negative values of V as the temperature T_{e2} dominates the probe response, and that this occurs even at small values of p . In fact, increasing p appears to cause the curve to asymptote a bit sooner (at a smaller negative voltage), but while that shifts the curve

higher on the $\ln(I_p + I_i)$ axis, it does further affect the slope beyond the initial shift owing to the introduction of the second temperature species, implying that the initial small increase in p has a large effect but further increases have a much smaller relative effect.

The suprathermal beam cases for $n_e=10^{19} \text{ m}^{-3}$, $T_{e1}=1 \text{ eV}$, $T_{e2}=0.1 \text{ eV}$, $v'=5 \text{ eV}$, and p varying from 0-1.5% exhibit a different trend relative to the two-temperature cases. The I - V curves in Fig. 4e are not as smooth in the -7 to -2 volt region where the voltage applied to the probe is in the neighborhood of the bump-on-the-tail at v' . The $\ln(I_p + I_i)$ versus V curves in Fig. 4f are even more interesting, significantly deviating from the Maxwellian $p=0$ case as the voltage is decreased, but then rejoining the Maxwellian curve around $V=10 \text{ V}$. Unlike the two-temperature distribution response where the entire distribution is globally affected, the suprathermal beam electrons only appear to affect the electron saturation current values and the probe current collection in the neighborhood of applied voltages that are close to v' . Furthermore, we increased the value of p beyond the 0.015 value shown and found that the spacing between curves in Fig. 4f cease to show further appreciable change much beyond what is shown. Like with the two-temperature distribution, this implies that the initial small increase in p has a large effect but further increases have a much smaller relative effect.

B. Double Langmuir Probe

The double Langmuir probe, shown schematically in Fig. 2b, consists of two single probes that are electrically connected by an adjustable voltage source that is used to apply a controllable differential voltage V_{d2} between the probes. As such, the symmetric double probe electrical configuration constrains it to operate in the regime where

$$I_{p1} + I_{p2} = 0 \quad (11a)$$

$$V_2 - V_1 = V_{d2} \quad (11b)$$

By continuity, we know that the current collected or emitted by either probe cannot exceed the ion saturation current I_i . For a Maxwellian distribution, the characteristic for each probe is often written as

$$I_{p1} = I_i \left(\exp \left[\frac{e(V_1 - \phi_f)}{k_b T_e} \right] - 1 \right) \quad (12a)$$

$$I_{p2} = I_i \left(\exp \left[\frac{e(V_2 - \phi_f)}{k_b T_e} \right] - 1 \right) \quad (12b)$$

Manipulation of these equations under the constraints on the applied voltage and the current collection allows us to write an I - V characteristic for the double probe as

$$I_{p2} = I_i \tanh \left(\frac{eV_{d2}}{2k_B T_e} \right) \quad (13a)$$

$$I_{p1} = -I_i \tanh \left(\frac{eV_{d2}}{2k_B T_e} \right) \quad (13b)$$

We note that Eqs. (13) are simply mirror images of each other. The I - V curve shown in Fig. 5a was calculated numerically is for a double probe in an equilibrium plasma by determining the current collected to the probe while scanning the value of V_{d2} under the constraint of Eq. (11a). The graph shown matches the analytical result given in Eq. (13a).

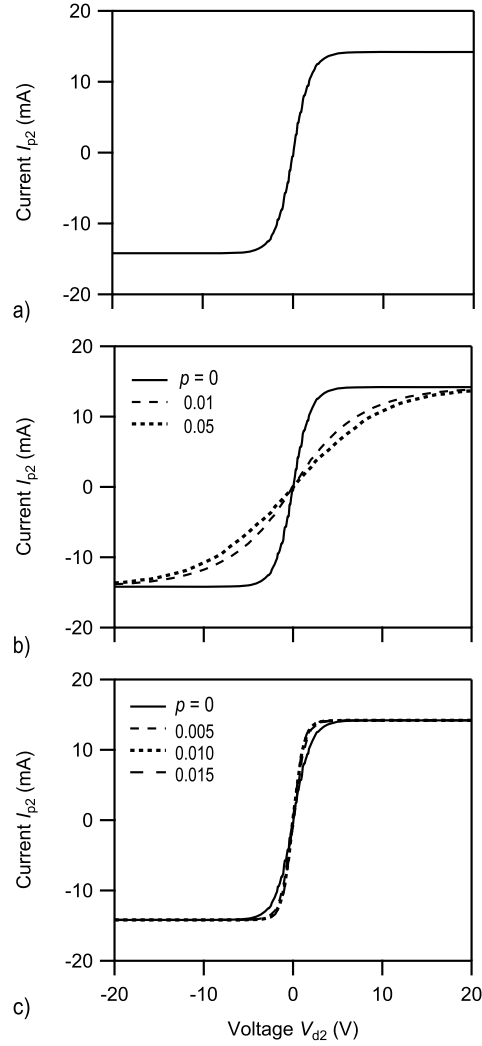


Figure 5. Double Langmuir probe I - V characteristics for a plasma with $n_e=10^{19} \text{ m}^{-3}$ calculated for a) a Maxwellian distribution with $T_e=1 \text{ eV}$, b) a two-temperature distribution with $T_{e1}=1 \text{ eV}$, $T_{e2}=5 \text{ eV}$, and p as given in the figure, and c) a suprathermal beam with $T_{e1}=1 \text{ eV}$, $T_{e2}=0.1 \text{ eV}$, $v'=5 \text{ eV}$, and p as given in the figure.

Continuing with the equilibrium assumption, the electron temperature can be found using Eq. (13a) by evaluating the slope at $V_{d2} = 0$ to yield

$$\frac{k_B T_e}{e} = I_i \left(2 \frac{dI_{p2}}{dV_{d2}} \Big|_{V_{d2}=0} \right)^{-1} \quad (14)$$

As with the single probe, this electron temperature and the measured ion saturation current can be used to calculate the number density n_e using Eq. (7). These were calculated and returned the values that were used to generate the initial equilibrium distribution function that was employed in the generation of the I - V curve.

If the distribution is non-equilibrium, the process of numerically generating the double probe I - V characteristic starts with the generation of a single probe I - V characteristic in the manner described in the previous section. The double probe characteristic can then be found by scanning the single probe characteristic for two values of I_p , corresponding to I_{p1} and I_{p2} , that sum to zero and are separated in voltage by V_{d2} . This is performed for the two-temperature distribution in Fig. 5b and for the suprathermal beam distribution in Fig. 5c. We note first that the two-temperature distribution and the suprathermal beam shift the I - V curve in different directions, with the former yielding a much more gradual zero voltage crossing as p is increased while the latter results in a much steeper crossing at $V_{d2}=0$. In Fig. 5b no values of p are shown beyond 0.05 because all greater values essentially lie on top of that value, indicating

that initial increases in p up to 0.05 have a large effect on the I - V characteristic but that additional increases in p have minimal additional impact. The values for p in Fig. 5c (other than 0) all essentially lie on top of each other, indicating that the shift introduced by the suprathermal beam occurs for a very small fraction of the plasma shifted into the beam, but that shifting additional particles into the beam beyond that initial change has little additional effect.

C. Triple Langmuir Probe

Unlike the single and double probes which are scanning in voltage to produce a characteristic I - V curve, the triple Langmuir probe operates at fixed voltage. As a consequence, the probe only samples certain discrete points on the single probe I - V curve as a function of time. The embedded assumption in the triple probe analysis is that the plasma has a Maxwellian electron energy distribution, and that the three points sampled on the single probe I - V curve are sufficient to extract electron temperature and number density data from the plasma. The examination of this assumption is the primary purpose of the present paper.

The triple probe can be operated in two distinct modes depending upon the nature of the electrical connections to the probe. The first, called the ‘current-mode’, is illustrated schematically in Fig. 6a. In this configuration, the constraints on the probe current and voltage are written as:

$$I_{p1} + I_{p2} + I_{p3} = 0 \quad (15a)$$

$$V_2 - V_1 = V_{d2} \quad (15b)$$

$$V_3 - V_1 = V_{d3} \quad (15c)$$

As with the double probe, when the voltages are specified these constraints correspond to a location on the single probe I - V curve for each probe tip. If the plasma has a Maxwellian electron distribution, then the current collected by each probe can be written as:

$$I_{p1} = i_e \exp\left(-\frac{e(-V_1)}{k_B T_e}\right) - I_i \quad (16a)$$

$$I_{p2} = i_e \exp\left(-\frac{e(-V_2)}{k_B T_e}\right) - I_i \quad (16b)$$

$$I_{p3} = i_e \exp\left(-\frac{e(-V_3)}{k_B T_e}\right) - I_i \quad (16c)$$

These equations can be manipulated to cancel the ion current term and to get the voltages in terms of V_{d2} and V_{d3} as

$$\frac{I_{p1} - I_{p2}}{I_{p1} - I_{p3}} = \frac{\exp\left(\frac{eV_1}{k_B T_e}\right) - \exp\left(\frac{eV_2}{k_B T_e}\right)}{\exp\left(\frac{eV_1}{k_B T_e}\right) - \exp\left(\frac{eV_3}{k_B T_e}\right)} = \frac{1 - \exp\left(\frac{eV_{d2}}{k_B T_e}\right)}{1 - \exp\left(\frac{eV_{d3}}{k_B T_e}\right)} \quad (17)$$

In triple Langmuir probe experiments, the values of V_{d2} and V_{d3} are fixed and measurements of the currents I_{p1} , I_{p2} , and I_{p3} plugged into Eq. (17) yield T_e . While the equation is transcendental, it is fairly straightforward to plug values of T_e into the right hand side of the equation to find a value that satisfies the ratio of currents on the left side. There are several ways in which the ion current can be estimated. In the present work we simply assumed that the ion saturation current was equal to I_{p3} and used that value in Eq. (7) to find n_e .

The triple probe can also be operated in ‘voltage-mode’, shown schematically in Fig. 6b. We don’t perform an analysis of this type of probe in the present work, but it is included for the sake of completeness. In this mode, probe 2 is at the floating potential and probes 1 and 3 operate as a double probe with the voltage between the two probes fixed. The constraints on a triple probe operating in this regime are

$$I_{p1} + I_{p3} = 0 \quad (18a)$$

$$I_{p2} = 0 \quad (18b)$$

$$\phi_f - V_1 = V_{d2} \quad (18c)$$

$$V_3 - V_1 = V_{d3} \quad (18d)$$

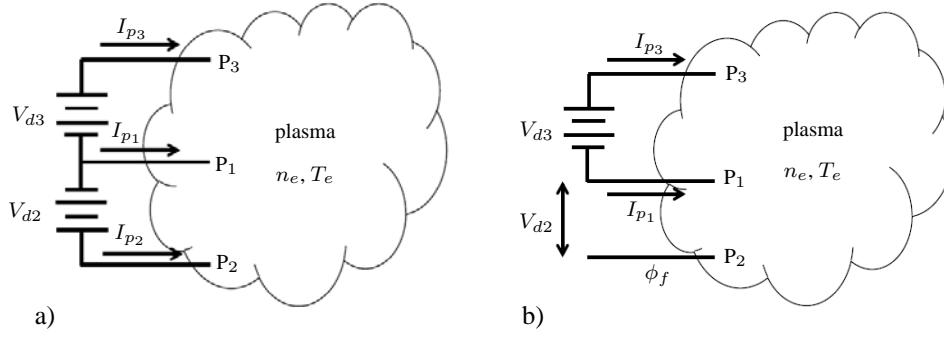


Figure 6. Schematic representation showing a) a current-mode triple Langmuir probe and b) a voltage-mode triple Langmuir probe.

If the plasma is Maxwellian, the results of Eq. (17) still hold. Using the constraints above, this can be simplified to

$$\frac{I_{p1} - I_{p2}}{I_{p1} - I_{p3}} = \frac{1}{2} = \frac{1 - \exp\left(\frac{eV_{d2}}{k_B T_e}\right)}{1 - \exp\left(\frac{eV_{d3}}{k_B T_e}\right)} \quad (19)$$

where the values of the collected currents has been eliminated from the equation. Since V_{d3} is fixed, finding T_e comes down to a measurement of V_{d2} . Once T_e is known, the value of n_e can again be found in the same manner employed by for the ‘current mode’ analysis.

IV. Triple Probe Analysis

In this section, we simulate the current collection characteristics of a triple probe operating in ‘current mode’ and inserted into a non-equilibrium plasma. The triple probe analysis given in the preceding section, which was derived for an equilibrium plasma condition, is employed to show the values of T_e and n_e that would be obtained. The voltage values used in the following analysis are $V_{d2}=3$ V and $V_{d3}=9$ V.

Electron temperatures and number densities found using the equilibrium analysis technique are presented for the two-temperature Maxwellian case in Fig 7. These data are generated for increasing values of p at temperatures T_{e2} of 3, 5, and 7 eV. The values at $p=0$ represent the Maxwellian equilibrium condition and corresponds to the input values of T_{e1} of 1 eV and n_e of 10^{19} m^{-3} . We observe that most of the change in these calculated values occurs for small increases in p ($\leq 5\%$), with the temperature quickly asymptoting to the value of T_{e2} and the calculated number density being severely depressed from the actual value. However, like with the single and double probe data, after the initial change as a function of increasing p , further increases in p appear to have little to no effect. Recall from the single probe data of Fig. 4d that the slope of the curves on the $\ln(I_p + I_i)$ plotted against V were primarily controlled by the higher temperature distribution, even at low values of p , and that increasing p had little effect on the slope, which is used to find the temperature of the distribution. It is, therefore, unsurprising that triple probe data increase very quickly as a function of p to equal the higher temperature second Maxwellian distribution. It is also unsurprising that the calculated number density is much lower than the actual value since the electron temperature appears in the denominator of Eq. (7) when it is re-cast to solve for n_e .

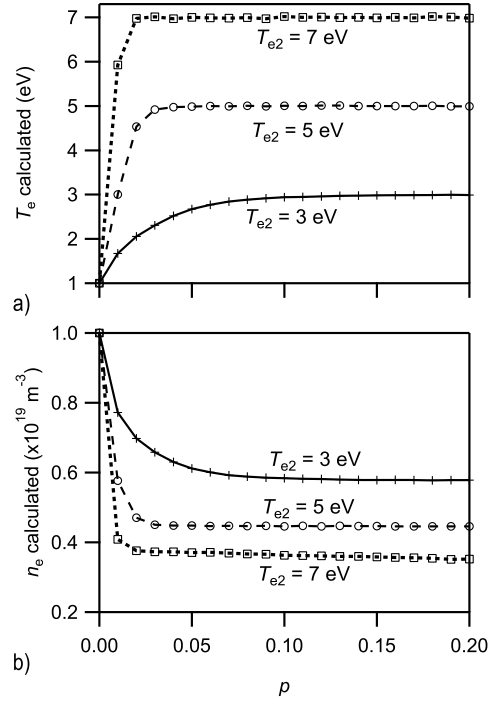


Figure 7. a) Electron temperature and b) number density that would be found using a triple probe measurement and equilibrium analysis on a two-temperature distribution with $n_e=10^{19} \text{ m}^{-3}$, $T_{e1}=1 \text{ eV}$, for given values of T_{e2} as a function of the fraction p .

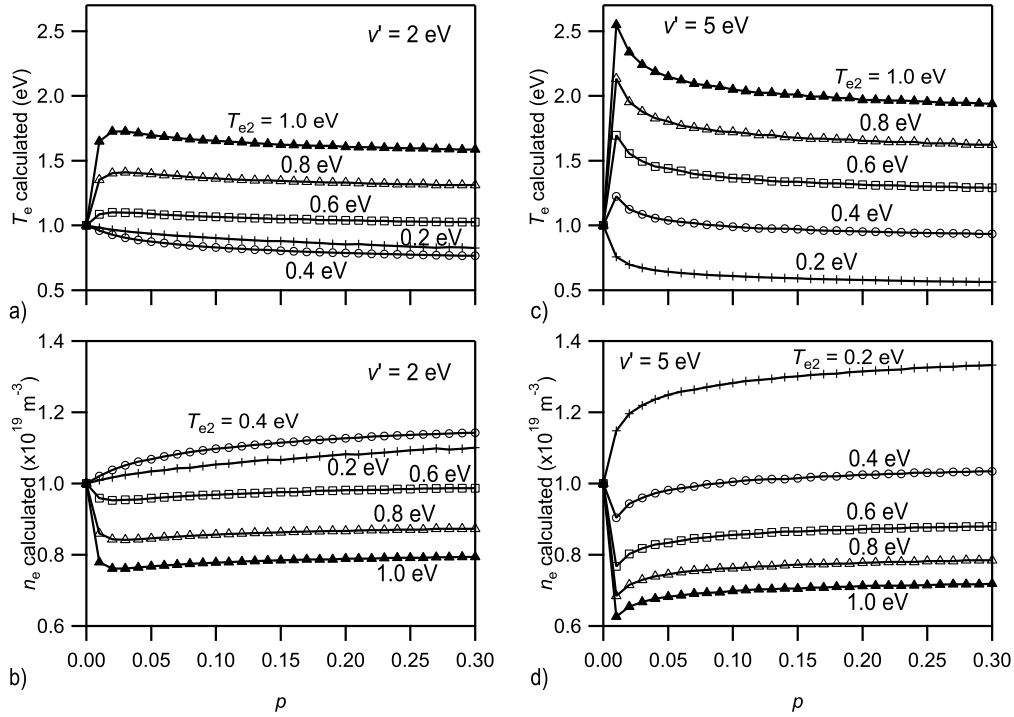


Figure 8. Electron temperature and number density that would be found using a triple probe measurement and equilibrium analysis on a suprathermal beam distribution with $n_e=10^{19} \text{ m}^{-3}$, $T_{e1}=1 \text{ eV}$, for given values of T_{e2} as a function of the fraction p for the offset v' of 2 eV (a and b) and 5 eV (c and d).

Triple probe data simulated for suprathermal cases are presented in Fig. 8. As before, the values at $p=0$ represent the Maxwellian equilibrium condition and corresponds to the input values of T_{e1} of 1 eV and n_e of 10^{19} m^{-3} . The temperature and number density for a shift v' of 2 eV are plotted in subplots a and b, while data for v' of 5 eV are found in c and d. Data are generated and analyzed for several values of T_{e2} and for increasing values of p . The magnitude of the shift from the equilibrium values increases with increasing v' , and for the most part it increases with increasing T_{e2} . The data in both cases first shift in one direction as a function of increasing T_{e2} , but as this value continues to grow the shift in computed temperature and number density reverses. The key takeaway is that most of the shift in the computed parameters occurs for small values of p , and increasing p further only yields limited additional deviation.

The data presented in this section demonstrate that a plasma having a non-Maxwellian electron distribution can significantly affect the results of a triple probe measurement and analysis undertaken assuming that the plasma has an equilibrium distribution. The deviation from the actual plasma properties found using this type of analysis is appreciably large when the plasma is shifted from the equilibrium distribution into a non-equilibrium distribution. Most of the shift in the computed plasma properties occurs for small values of p , implying that the triple probe analysis technique is sensitive to even small deviations from a Maxwellian distribution, which yield a profound impact on the temperature and number density data such measurements would otherwise report.

V. Conclusions

We have conducted a numerical study analyzing the response of Langmuir probes performing measurements on representative non-equilibrium plasmas. While an experimentalist using a single probe can use the data generated to determine if the plasma is in an equilibrium configuration, the unswept voltages of the triple probe do not permit such a determination. In fact, the triple probe tips only sample current values corresponding to three closely-spaced, discrete points on the single Langmuir probe I - V characteristic, with these data then analyzed under the assumption of equilibrium. Triple probe current collection data generated for two-temperature Maxwellian and suprathermal beam non-equilibrium plasma distributions show that the calculated electron temperature and number density deviate significantly from the input values when the triple probe data are analyzed under the assumption of an equilibrium plasma. In the two-temperature Maxwellian case, the computed temperature is controlled by and approaches the temperature of the hotter species. In both cases, most of the computed shift from the equilibrium values occurs for very small values of p ($\leq 5\%$), strongly implying that the triple probe measurements are extremely sensitive to small deviations from equilibrium. Beyond this initial shift, the computed plasma properties change much less as p is further increased.

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